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YGB - MMS PAPER J - QUESTION 1

$$\begin{aligned}\text{a) I)} \quad & \sqrt{108} + \sqrt{3} \\ &= \sqrt{36 \times 3} + \sqrt{3} \\ &= 6\sqrt{3} + \sqrt{3} \\ &= \underline{\underline{7\sqrt{3}}}\end{aligned}$$

$$\begin{aligned}\text{II)} \quad & \frac{\sqrt{6} + \sqrt{3}}{\sqrt{2} + 1} = \frac{(\sqrt{6} + \sqrt{3})(\sqrt{2} - 1)}{(\sqrt{2} + 1)(\sqrt{2} - 1)} \\ &= \frac{\sqrt{12} - \sqrt{6} + \sqrt{6} - \sqrt{3}}{2 - \sqrt{2} + \sqrt{2} - 1} \\ &= \frac{\sqrt{12} - \sqrt{3}}{1} \\ &= \sqrt{4} \sqrt{3} - \sqrt{3} \\ &= 2\sqrt{3} - \sqrt{3} \\ &= \underline{\underline{\sqrt{3}}}\end{aligned}$$

ALTERNATIVE FOR a II

$$\frac{\sqrt{6} + \sqrt{3}}{\sqrt{2} + 1} = \frac{\sqrt{3}\sqrt{2} + \sqrt{3}}{\sqrt{2} + 1} = \frac{\sqrt{3}(\sqrt{2} + 1)}{\sqrt{2} + 1} = \underline{\underline{\sqrt{3}}}$$

A BGRF

b) PROCEED AS FOLLOWS

$$\begin{aligned}\Rightarrow & (5-x)^{\frac{3}{2}} = 8 \\ \Rightarrow & \left[(5-x)^{\frac{3}{2}}\right]^{\frac{2}{3}} = 8^{\frac{2}{3}} \\ \Rightarrow & (5-x)^1 = (\sqrt[3]{8})^2 \\ \Rightarrow & 5-x = 4 \\ \Rightarrow & \underline{\underline{x=1}}\end{aligned}$$

ALTERNATIVE METHOD

$$\begin{aligned}\Rightarrow & (5-x)^{\frac{3}{2}} = 8 \\ \Rightarrow & \left[\sqrt[3]{5-x}\right]^3 = 8 \\ \Rightarrow & \sqrt{5-x} = \sqrt[3]{8} \\ \Rightarrow & \sqrt{5-x} = 2 \\ \Rightarrow & 5-x = 4 \\ \Rightarrow & \underline{\underline{x=1}}\end{aligned}$$

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1XGB - MPI PAPER J - QUESTION 2

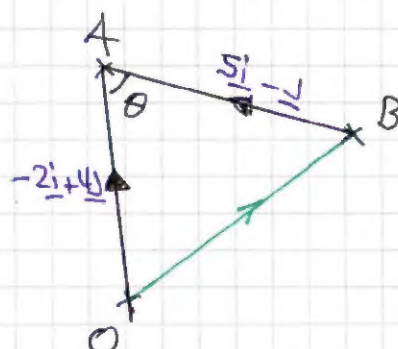
a) START WITH A DIAGRAM

$$\vec{OB} = \vec{OA} + \vec{AB}$$

$$\vec{OB} = \vec{OA} - \vec{BA}$$

$$\vec{OB} = (-2\hat{i} + 4\hat{j}) - (5\hat{i} - \hat{j})$$

$$\vec{OB} = -7\hat{i} + 5\hat{j}$$



FINDING THE DISTANCE OB

$$|\vec{OB}| = |-7\hat{i} + 5\hat{j}| = \sqrt{(-7)^2 + 5^2} = \sqrt{74} \approx 8.60$$

b) FIND THE LENGTHS OF AB & OA

$$|\vec{OA}| = |-2\hat{i} + 4\hat{j}| = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20}$$

$$|\vec{BA}| = |5\hat{i} - \hat{j}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$$

BY THE COSINE RULE

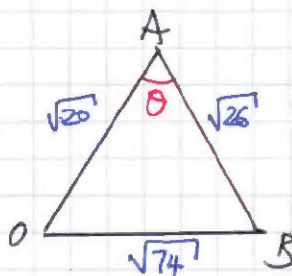
$$|\vec{OB}|^2 = |\vec{OA}|^2 + |\vec{AB}|^2 - 2|\vec{OA}||\vec{AB}|\cos\theta$$

$$74 = 20 + 26 - 2\sqrt{20}\sqrt{26}\cos\theta$$

$$2\sqrt{20}\sqrt{26}\cos\theta = -28$$

$$\cos\theta = -0.61394...$$

$$\theta \approx 128^\circ$$



1YGB - PAPER J - QUESTION 3

WRITE IN INDEX NOTATION

$$y = \sqrt{2x} = \sqrt{2} x^{\frac{1}{2}}$$

DIFFERENTIATING W.R.T x & FINDING THE GRADIENT AT $x=2$

$$\frac{dy}{dx} = \frac{1}{2} \sqrt{2} x^{-\frac{1}{2}}$$

$$\left. \frac{dy}{dx} \right|_{x=2} = \frac{1}{2} \sqrt{2} \times 2^{-\frac{1}{2}} = \frac{1}{2} \cancel{\sqrt{2}} \times \frac{1}{\cancel{\sqrt{2}}} = \frac{1}{2}$$

NORMAL GRADIENT IS -2

WHEN $x=2$, $y = \sqrt{2 \times 2} = 2$ IE (2,2)

EQUATION OF NORMAL SATISFIES

$$y - y_0 = m(x - x_0)$$

$$y - 2 = -2(x - 2)$$

$$y - 2 = -2x + 4$$

$$\underline{y = -2x + 6}$$

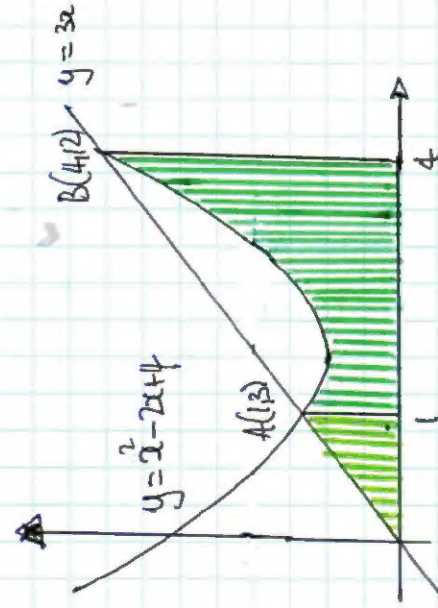
YGB-NP1 PAPER 3 - QUESTION 4

START BY FINDING THE POINTS OF INTERSECTION

$$\left. \begin{array}{l} y = x^2 - 2x + 4 \\ y = 3x \end{array} \right\} \Rightarrow \begin{array}{l} x^2 - 2x + 4 = 3x \\ \Rightarrow x^2 - 5x + 4 = 0 \\ \Rightarrow (x-1)(x-4) = 0 \\ \Rightarrow x = 1 \quad y = 3 \\ \quad \quad \quad x = 4 \quad y = 12 \end{array}$$

$$\therefore A(1,3) \quad \& \quad B(4,12)$$

LOOKING AT THE DIAGRAM BELOW



$$\text{AREA OF TRIANGLE} = \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$$

(LIGHT GREEN)

$$\begin{aligned} \text{AREA UNDER THE CURVE} &= \int_1^4 (x^2 - 2x + 4) \, dx \\ &= \left[\frac{1}{3}x^3 - x^2 + 4x \right]_1^4 \\ &= \left(\frac{64}{3} - 16 + 16 \right) - \left(\frac{1}{3} - 1 + 4 \right) \\ &= \frac{64}{3} - \frac{1}{3} + 1 - 4 \\ &= 18 \end{aligned}$$

(DARK GREEN)

$$\text{REQUIRED AREA} = 18 + \frac{3}{2} = \frac{39}{2}$$

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1YGB - MPI - PAPER J - QUESTIONS

USING THE IDENTITY $\cos^2 A + \sin^2 A \equiv 1$

$$\Rightarrow 3\cos^2 2\phi - 4\sin^2 2\phi = 15\cos 2\phi - 6$$

$$\Rightarrow 3\cos^2 2\phi - 4(1 - \cos^2 2\phi) = 15\cos 2\phi - 6$$

$$\Rightarrow 3\cos^2 2\phi - 4 + 4\cos^2 2\phi = 15\cos 2\phi - 6$$

$$\Rightarrow 7\cos^2 2\phi - 15\cos 2\phi + 2 = 0$$

FACTORIZING OR USE THE QUADRATIC FORMULA

$$\Rightarrow (7\cos 2\phi - 1)(\cos 2\phi - 2) = 0$$

$$\Rightarrow \cos 2\phi = \begin{cases} \frac{1}{7} \\ \cancel{2} \end{cases} \quad -1 \leq \cos 2\phi \leq 1$$

$$\arccos\left(\frac{1}{7}\right) = 81.7867^\circ \dots$$

$$\begin{cases} 2\phi = 81.7867^\circ \pm 360n \\ 2\phi = 278.213^\circ \pm 360n \end{cases} \quad n=0,1,2,3,\dots$$

$$\begin{cases} \phi = 40.9^\circ \pm 180n \\ \phi = 139.1^\circ \pm 180n \end{cases}$$

IN THE RANGE GIVEN

$$\underline{\phi = 40.9^\circ, 139.1^\circ, 220.9^\circ, 319.1^\circ}$$

YGB-MPI PAPER J - QUESTION 6

$$\text{let } y = f(x) = \frac{1}{x}$$

$$f(x+h) = \frac{1}{x+h}$$

$$\underline{f(x+h) - f(x)} = \frac{1}{x+h} - \frac{1}{x} = \frac{x - (x+h)}{x(x+h)} = \frac{-h}{x(x+h)}$$

USING THE FORMAL DEFINITION OF THE DERIVATIVE

$$\frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left[\frac{f(x+h) - f(x)}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\frac{-h}{x(x+h)}}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{-h}{x(x+h)} \div h \right]$$

$$= \lim_{h \rightarrow 0} \left[\frac{\cancel{-h}}{x^2 + xh} \times \frac{1}{h} \right]$$

$$= \lim_{h \rightarrow 0} \left[-\frac{1}{x^2 + xh} \right]$$

TAKING LIMITS, AS $h \rightarrow 0$

$$= -\frac{1}{x^2 + xh}$$

$$= -\frac{1}{x^2} \quad \text{AS REQUIRED}$$

YGB - MPI PAPER J - QUESTION 7

a) USING EACH OF THE POINTS IN TURN, FORM TWO EQUATIONS IN a & b

$$\begin{aligned} (-4, 7) &\Rightarrow (-4)^2 + 7^2 + a(-4) + b \times 7 + 43 = 0 \\ (-2, 5) &\Rightarrow (-2)^2 + 5^2 + a(-2) + b \times 5 + 43 = 0 \end{aligned} \quad \Rightarrow$$

$$\begin{aligned} 16 + 49 - 4a + 7b + 43 &= 0 \\ 4 + 25 - 2a + 5b + 43 &= 0 \end{aligned} \quad \Rightarrow$$

$$\begin{aligned} \times (-1) &\rightarrow -4a + 7b = -108 \\ \times 2 &\rightarrow -2a + 5b = -72 \end{aligned} \quad \Rightarrow$$

$$\begin{aligned} 4a - 7b &= 108 \\ -4a + 10b &= -144 \end{aligned} \quad \Rightarrow$$

$$\begin{aligned} \text{ADDING} \quad 3b &= -36 \\ b &= -12 \end{aligned}$$

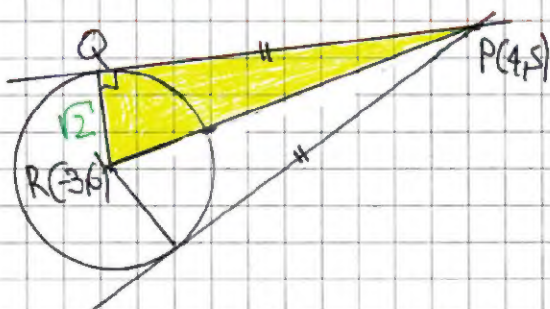
$$\begin{aligned} \& \quad 4a - 7b &= 108 \\ 4a + 84 &= 108 \\ 4a &= 24 \\ a &= 6 \end{aligned}$$

THENCE WE NOW HAVE

$$\begin{aligned} x^2 + 6x + y^2 - 12y + 43 &= 0 \\ (x+3)^2 - 9 + (y-6)^2 - 36 + 43 &= 0 \\ (x+3)^2 + (y-6)^2 &= 2 \end{aligned}$$

$$\therefore \text{CENTRE } (-3, 6), \text{ RADIUS} = \sqrt{2}$$

b) LOOKING AT THE DIAGRAM BELOW



$$P(4, 5) \quad R(-3, 6)$$

$$|PR| = \sqrt{(-3-4)^2 + (6-5)^2}$$

$$|PR| = \sqrt{(-7)^2 + 1^2}$$

$$|PR| = \sqrt{50}$$

1YGB - NPI PAPER J - QUESTION 7

FINALLY BY PYTHAGORAS

$$\Rightarrow |PQ|^2 + |QR|^2 = |PR|^2$$

$$\Rightarrow |PQ|^2 + (\sqrt{2})^2 = (\sqrt{50})^2$$

$$\Rightarrow |PQ|^2 + 2 = 50$$

$$\Rightarrow |PQ|^2 = 48$$

$$\Rightarrow |PQ| = \sqrt{48}$$

$$\Rightarrow \underline{\underline{|PQ| = 4\sqrt{3}}}$$

As required

1YGB - MPI PAPER J - QUESTION 8

EXPANDING USING THE STANDARD BINOMIAL FORMULA

$$f(x) = (3-2x)^2 (1+2x)^6$$

$$f(x) = (9-12x+4x^2) \left[1 + \frac{6}{1}(2x)^1 + \frac{6 \times 5}{1 \times 2}(2x)^2 + \frac{6 \times 5 \times 4}{1 \times 2 \times 3}(2x)^3 + \dots \right]$$

$$f(x) = (9-12x+4x^2)(1+12x+60x^2+160x^3+\dots)$$

$$\begin{array}{rcl} f(x) = & 9 + 108x + 540x^2 + 1440x^3 + \dots & \\ & - 12x - 144x^2 - 720x^3 + \dots & \\ & + 4x^2 + 48x^3 + \dots & \end{array}$$

$$\underline{f(x) = 9 + 96x + 400x^2 + 768x^3 + \dots}$$

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1YGB - MPI Paper 5 - Question 9

a) Factorize by common factor then the quadratic

$$y = 5x - 4x^2 - x^3$$


$$y = -x^3 - 4x^2 + 5x$$

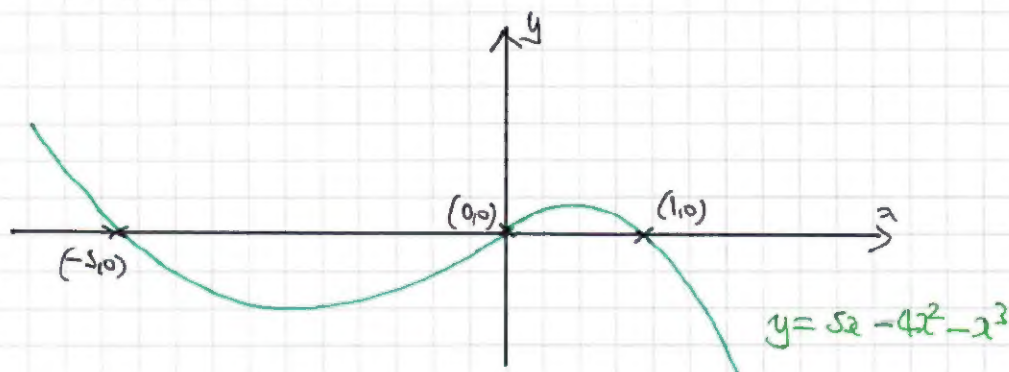
$$y = -x(x^2 + 4x - 5)$$

$$y = -x(x-1)(x+5)$$

or $y = x(1-x)(x+5)$

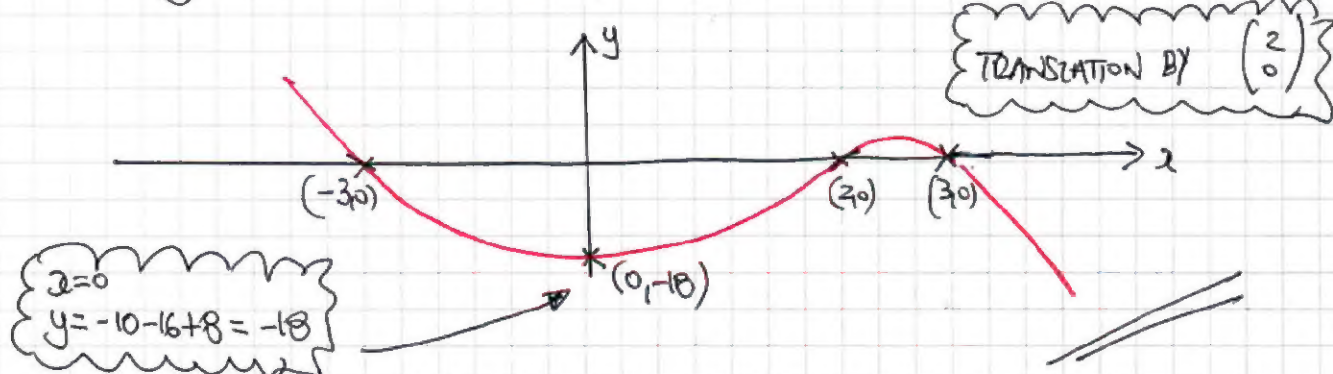
b) Collect the information & sketch

- $-x^3 \Rightarrow$ 
- $x=0 \Rightarrow y=0$ (0,0)
- $y=0 \Rightarrow x = \begin{matrix} 0 & (0,0) \\ 1 & (1,0) \\ -5 & (-5,0) \end{matrix}$



c) Looking at 'transformations'

if $y = f(x) = 5x - 4x^2 - x^3$, then $f(x-2) = 5(x-2) - 4(x-2)^2 - (x-2)^3$



IYGB - MPI PAPER J - QUESTION 10

PROCEED AS FOLLOWS

$$\Rightarrow 3kx^2 - 2kx - 4x + 3 = 0$$

$$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$$

$$\Rightarrow 3kx^2 - (2k+4)x + 3 = 0$$

TWO DIFFERENT REAL ROOTS $b^2 - 4ac > 0$

$$\Rightarrow [-(2k+4)]^2 - 4 \times 3k \times 3 > 0$$

$$\Rightarrow (2k+4)^2 - 36k > 0$$

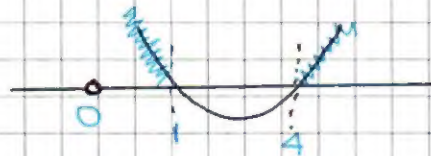
$$\Rightarrow 4k^2 + 16k + 16 - 36k > 0$$

$$\Rightarrow 4k^2 - 20k + 16 > 0$$

$$\Rightarrow k^2 - 5k + 4 > 0$$

$$\Rightarrow (k-4)(k-1) > 0$$

CRITICAL VALUES



$$\therefore \underline{k < 1, k \neq 0 \text{ OR } k > 4}$$

NOTE THAT IF $k=0$, THE QUADRATIC SEEMS TO EXIST & REDUCES TO A LINEAR EQUATION WITH ONE ROOT ONLY

1YGB - MPI PART J - QUESTION 11

PROCEED AS FOLLOWS

$$\Rightarrow \ln(x+1)^2 - 2y = 2$$

$$\Rightarrow 2\ln(x+1) - 2y = 2$$

$$\Rightarrow \ln(x+1) - y = 1$$

AND

$$x + e^y = 1$$

$$\Rightarrow e^y = 1 - x$$

$$\Rightarrow y = \ln(1-x)$$

$$\ln(x+1) - \ln(1-x) = 1$$

$$\Rightarrow \ln\left(\frac{x+1}{1-x}\right) = 1$$

$$\Rightarrow \frac{x+1}{1-x} = e^1$$

$$\Rightarrow \frac{x+1}{1-x} = e$$

$$\Rightarrow x+1 = e - ex$$

$$\Rightarrow x + ex = e - 1$$

$$\Rightarrow x(1+e) = e - 1$$

$$\Rightarrow x = \frac{e-1}{e+1}$$

$$\text{AND } y = \ln(1-x)$$

$$y = \ln\left(1 - \frac{e-1}{e+1}\right)$$

$$y = \ln\left(\frac{\cancel{e+1} - e + 1}{e+1}\right)$$

$$y = \ln\left(\frac{2}{e+1}\right)$$

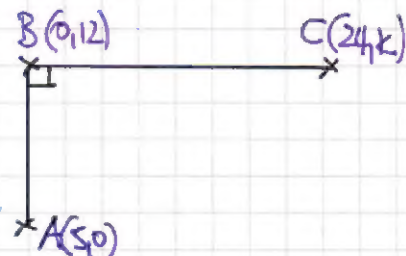
$$\therefore (x, y) = \left[\frac{e-1}{e+1}, \ln\left(\frac{2}{e+1}\right) \right]$$

1YGB - MPI PART J - QUESTION 12

a) WORK WITH GRADIENTS FIRST

$$\text{GRADIENT AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 0}{0 - 5} = -\frac{12}{5}$$

$$\text{GRADIENT BC} = \frac{k - 12}{24 - 0} = \frac{k - 12}{24}$$



NOW AS $AB \perp BC$, $\text{GRAD BC} = +\frac{5}{12}$

$$\Rightarrow \frac{k - 12}{24} = \frac{5}{12}$$

$$\Rightarrow 12k - 144 = 120$$

$$\Rightarrow 12k = 264$$

$$\Rightarrow \underline{k = 22} \quad \text{As Required}$$

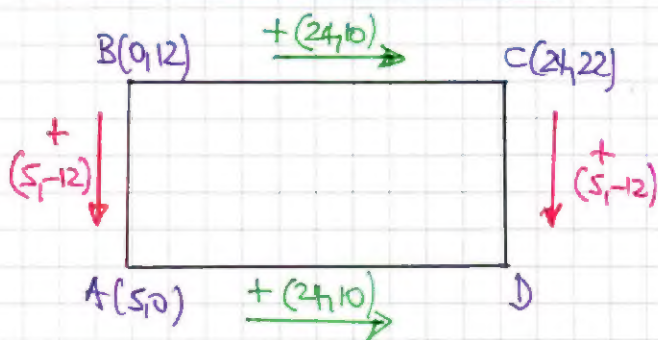
NEXT THE LENGTHS, USING $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$|AB| = \sqrt{(12 - 0)^2 + (0 - 5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13$$

$$|BC| = \sqrt{(22 - 12)^2 + (24 - 0)^2} = \sqrt{100 + 576} = \sqrt{676} = 26$$

$$\therefore \text{AREA} = 13 \times 26 = \underline{338}$$

b) BY INSPECTION, OR "VECTORS"



$$\therefore \underline{D(24,10)}$$

LYGB - MPI PAPER J - QUESTION 13

a) COMPLETING THE SQUARE

$$y = 4x^2 + 24x + A$$

$$y = 4 \left[x^2 + 6x + \frac{A}{4} \right]$$

$$y = 4 \left[(x+3)^2 - 9 + \frac{A}{4} \right]$$

$$y = 4(x+3)^2 - 36 + A$$

$$\begin{aligned} \text{If } p &= 4 \\ q &= 3 \\ r &= 4 - 36 \end{aligned}$$

b) SOLVING SIMULTANEOUSLY

$$y = 4x^2 + 24x + A$$

$$y = Bx + 10$$

$$\Rightarrow 4x^2 + 24x + A = Bx + 10$$

$$\Rightarrow 4x^2 + (24-B)x + (A-10) = 0$$

If $x = -1$

$$4 - (24-B) + (A-10) = 0$$

$$4 - 24 + B + A - 10 = 0$$

$$A + B = 30$$

$$B = 30 - A$$

If $x = -\frac{21}{4}$

$$4\left(-\frac{21}{4}\right)^2 + (24-B)\left(-\frac{21}{4}\right) + (A-10) = 0$$

$$\frac{441}{4} - 126 + \frac{21}{4}B + A - 10 = 0$$

$$441 - 504 + 21B + 4A - 40 = 0$$

$$4A + 21B = 103$$

$$4A + 21(30-A) = 103$$

$$4A + 630 - 21A = 103$$

$$527 = 17A$$

$$A = 31$$

$$B = -1$$